A Study of Index Correlations.

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A problem of some importance in Medical Statistics is of the following nature. In a series of districts with population z_0, z_1, z_2, &c., the deaths from a certain disease are x_0, x_1, x_2, &c., and from some other disease, y_0, y_1, y_2, &c., is there any association between the x's and y's which is independent of the common relation of each with z? Assuming that the question of differences in age constitution does not arise, it would appear at first sight that all we require is either ${}_zr_{xy}$ or ${}_zr_{\frac{x}{z}\frac{y}{z}}$ and that these constants should not differ in value. Thus, in a recent note, Professor Karl Pearson writes:—"Now it is easy to show that the correlation of $\frac{\delta}{p}$ and $\frac{\delta'}{p}$ for p constant is precisely the same thing as the correlation of δ and δ' for p constant."* The δ and δ' of this quotation are our x and y and the p is our z. The proof is as follows:—

Looking at the problem from the standpoint of algebra, we have:—

$$ {}_zr_{\frac{x}{z}\frac{y}{z}} = \frac{r_{\frac{x}{z}\frac{y}{z}} - r_{\frac{x}{z}z}\, r_{\frac{y}{z}z}}{\sqrt{\{(1 - r^2_{\frac{x}{z}z})(1 - r^2_{\frac{y}{z}z})\}}} \tag{1} $$

But, if deviations are small compared with the mean:—†

$$ r_{\frac{x}{z}\frac{y}{z}} = \frac{\frac{\sigma_x}{\bar{x}}\frac{\sigma_y}{\bar{y}} r_{xy} - \frac{\sigma_x}{\bar{x}}\frac{\sigma_z}{\bar{z}} r_{xz} - \frac{\sigma_y}{\bar{y}}\frac{\sigma_z}{\bar{z}} r_{yz} + \frac{\sigma^2_z}{\bar{z}^2}}{\sqrt{\left\{\left(\frac{\sigma^2_x}{\bar{x}^2} + \frac{\sigma^2_z}{\bar{z}^2} - 2\frac{\sigma_x}{\bar{x}}\frac{\sigma_z}{\bar{z}} r_{xz}\right)\left(\frac{\sigma^2_y}{\bar{y}^2} + \frac{\sigma^2_z}{\bar{z}^2} - 2\frac{\sigma_y}{\bar{y}}\frac{\sigma_z}{\bar{z}} r_{yz}\right)\right\}}}, \tag{2} $$

$$ r_{\frac{x}{z}z} = \frac{\frac{\sigma_x}{\bar{x}} r_{xz} - \frac{\sigma_z}{\bar{z}}}{\sqrt{\left(\frac{\sigma^2_x}{\bar{x}^2} - 2\frac{\sigma_x}{\bar{x}}\frac{\sigma_z}{\bar{z}} r_{xz} + \frac{\sigma^2_z}{\bar{z}^2}\right)}}, \tag{3} $$

$$ r_{\frac{y}{z}z} = \frac{\frac{\sigma_y}{\bar{y}} r_{yz} - \frac{\sigma_z}{\bar{z}}}{\sqrt{\left(\frac{\sigma^2_y}{\bar{y}^2} - 2\frac{\sigma_y}{\bar{y}}\frac{\sigma_z}{\bar{z}} r_{yz} + \frac{\sigma^2_z}{\bar{z}^2}\right)}} \tag{4} $$

Substituting these values in (1) and reducing, we obtain:—

$$ \frac{r_{xy} - r_{xz}\, r_{yz}}{\sqrt{\{(1 - r^2_{xz})(1 - r^2_{yz})\}}} \tag{5} $$

* *Journal of the Royal Statistical Society*, LXXIII, 1910, p. 536.

† Pearson, *Proc. Roy. Soc.* lx, 1897, p. 489.

which is ${}_{z}r_{xy}$. In precisely the same way we can show that ${}_{\frac{1}{z}}r_{\frac{x}{z}\frac{y}{z}} = {}_{z}r_{xy}$. These results depend on the assumption that in such expansions as $\left(1 + \frac{\epsilon_x}{\bar{x}}\right)^{-1}$ (where ϵ_x is a variation from the mean, $\bar{x}$.) terms beyond the third may be neglected. Let us express ${}_{z}r_{\frac{x}{z}\frac{y}{z}}$ in terms of product moments, using a method devised by Pearson for the study of index frequencies.*

Put $z = \frac{1}{w}$, let p denote an xw moment coefficient about zero, p' a yw moment cofficient about zero, and P a moment coefficient involving an (xw) or (yw) *i.e.*, $(xw)\ w$ or $w\ (yw)$ or $(xw)(yw)$. A symbol with a line above it denotes the mean value of the symbol and the subscript numerals denote the order of the product, *i.e.*, $p_{22} = \frac{\mathrm{S}x^2w^2}{\mathrm{N}}$.

We have—

$$(\bar{x}\bar{w}) = p_{11},\ (\bar{y}\bar{w}) = p'_{11}$$

$$\sigma^2_{xw} = p_{22} - p^2_{11},\ \sigma^2_{yw} = p'_{22} - p'^2_{11}.$$

$$r_{(xw)(yw)} = \frac{\mathrm{P}_{(xw)(yw)} - p_{11}p'_{11}}{\sqrt{\{(p_{22} - p^2_{11})(p'_{22} - p'^2_{11})\}}} \qquad r_{(xw)w} = \frac{\mathrm{P}_{(xw)w} - \bar{w}p_{11}}{\sqrt{\{\sigma^2_w(p_{22} - p^2_{11})\}}}$$

$$r_{(yw)w} = \frac{\mathrm{P}_{(yw)w} - \bar{w}p'_{11}}{\sqrt{\{\sigma^2_w(p'_{22} - p'^2_{11})\}}}$$

hence—

$${}_{w}r_{(xw)(yw)} = \frac{\sigma^2_w(\mathrm{P}_{(xw)(yw)} - p_{11}p'_{11}) - (\mathrm{P}_{(wx)w} - \bar{w}p_{11})(\mathrm{P}_{(yw)w} - \bar{w}p'_{11})}{[\{\sigma^2_w(p_{22} - p^2_{11}) - (\mathrm{P}_{(wx)w} - \bar{w}p_{11})^2\}\ \{\sigma^2_w(p'_{22} - p'_{11}) - (\mathrm{P}_{(wy)w} - \bar{w}p'_{11})^2\}]^{\frac{1}{2}}} \quad (6)$$

This may be verified by referring the product moments to the means and dividing out. The expression evidently becomes:

$$\frac{r_{(xw)(yw)} - r_{(wx)w} \cdot r_{(wy)w}}{\sqrt{\{(1 - r^2_{(wx)w})(1 - r^2_{(wy)w})\}}} \quad (7)$$

From what has already been proved we know that this is equal to ${}_{w}r_{xy}$ or ${}_{\frac{1}{w}}r_{xy} = {}_{z}r_{xy}$ provided deviations from the means are sufficiently small to admit of our using (2), (3) and (4).

Should this condition not be fulfilled, the further reduction of (6) or (7) must depend on the nature of the regression equations connecting x, y and w. If we assume $(w - \bar{w}) = \mathrm{R}_1\ (x - \bar{x}) + \mathrm{R}_2\ (y - \bar{y})$, (6) can be expressed in terms of product moments, involving x and y, and of the regression coefficients, but such an assumption would not be, in general, compatible with a linear relation connecting x, y and $\frac{1}{w}$. There is accordingly no reason to expect that ${}_{z}r_{xy}$ will generally be the same as ${}_{z}r_{\frac{x}{z}\frac{y}{z}}$.

* *Idem, Biometrika*, vii, 1909-10, p. 531.

In Table 1 are collected examples of ${}_zr_{xy}$ and ${}_zr_{\frac{x}{z}\frac{y}{z}}$ which we have worked out. With regard to the material upon which this and several other tables are based, we may remark that it was not collected for the purpose of this paper and that its actual significance will be discussed elsewhere. The present paper is exclusively devoted to a question of method. We have purposely excluded the consideration of the question of age influence as not relevant to the subject; in the table, however, some values of ${}_zr_{xy}$ and ${}_zr_{\frac{x}{z}\frac{y}{z}}$ are given for data which have been corrected for age distribution.

An inspection of the table suggests certain remarks. Thus in some cases the differences are large, although not necessarily significant, and in most cases ${}_zr_{xy}$ is larger than ${}_zr_{\frac{x}{z}\frac{y}{z}}$. Another point to be noted is the fact that ${}_zr_{\frac{x}{z}\frac{y}{z}}$ rarely differs significantly from $r_{\frac{x}{z}\frac{y}{z}}$. The practical importance of this is that, were we satisfied that ${}_zr_{\frac{x}{z}\frac{y}{z}}$ was the correct constant to employ, our turn would be served by $r_{\frac{x}{z}\frac{y}{z}}$ and much labour would be saved. Having demonstrated the large discrepancies which may exist between the values of ${}_zr_{xy}$ and ${}_zr_{\frac{x}{z}\frac{y}{z}}$ we may state the object of this communication as being:—(1) to ascertain which coefficient should be employed in ordinary practice, (2) to discover if possible the source of the discrepancy.

In the first place it is to be observed that the actual numerical value of ${}_zr_{xy}$ is much more influenced by the presence in a series of data of a few very large absolute values than is ${}_zr_{\frac{x}{z}\frac{y}{z}}$.

This statement, which mainly applies when the number of observations is small and the coefficient of variation large and greatly influenced by the presence of certain values, could be expressed symbolically, but it seems better to proceed at once to arithmetical illustrations. To exhibit the effect in an exaggerated form we give an imaginary case (Table 2). We then pass to cases which actually have arisen or might arise in practice (Table 3).

The import of these results deserves rather careful consideration, and the following train of ideas at once presents itself. We find that ${}_zr_{xy}$ is much more sensitive than ${}_zr_{\frac{x}{z}\frac{y}{z}}$ to the introduction of data differing greatly from the original material in absolute measurements. But we know that the mixing of heterogeneous records having entirely different mean values leads to the production of correlations which are "spurious" and do not measure any real association between the variables. Consequently, it may perhaps be said, a method which is sensitive to such effects is much superior to one that registers them far less plainly. In other words, the use of ${}_zr_{xy}$ will put us on our guard against spurious correlation due to mixture, since we can check our results by dividing the data into roughly homogeneous series and recalculating the constants.

We have no doubt that these remarks will commend themselves to many statisticians, but we are ourselves unable to admit their sufficiency. We must remember that the word heterogeneity has no absolute significance, a series may be heterogeneous from one point of view and perfectly homogeneous when examined from another standpoint. If we mix together a number of millionaires and a sample of general labourers, the mixture may be perfectly homogeneous in respect of racial type, stature, age, weight, although wildly heterogeneous in respect of weekly income. The sample might be a perfectly appropriate one for the determination of stature and weight correlations, however inappropriate for the study of the correlation between either variable and income. In our actual problem it does not seem to us that a method which is greatly influenced by absolute variations in population should necessarily be superior to one not so influenced. Always provided that the absolute size of any population in the series is such that a ratio based thereon is not fundamentally unreliable, we do not see why our results should be in effect weighted by size or that an observation based on ten thousand inhabitants should tell less than one based on a million. Of course, in some problems this would not be true, but we contend that as a general proposition we have no right always to regard absolute magnitude as an element to be taken into account. The problem now under discussion is cognate with that considered by Mr. Yule in a recent paper.* Without necessarily assenting to all his conclusions, we should be disposed to think that the relative constancy of ${}_zr_{\frac{x}{z}\frac{y}{z}}$ is an argument in favour of its use in preference to that of ${}_zr_{xy}$ in the short series mostly available for work on the correlation of death and morbidity returns, where the size and even the sign of ${}_zr_{xy}$ may be determined, in extreme cases, by a single observation.

We must now look somewhat more closely into the causes of the discrepancy between ${}_zr_{xy}$ and ${}_zr_{\frac{x}{z}\frac{y}{z}}$. In the case of absolute numbers where each observation is weighted by its actual size, the presence of one large value may determine to a considerable extent the slope of the regression surface. When dealing with indices the weights of the different observations are approximately the same, owing to the fact that the correlations between x and z and y and z are positive and high, and that neither x nor y can ever be greater than z, so that in this case no single observation can have any predominating effect in determining the slope of the regression surface.

It is true that the sizes of the different populations vary to the same extent as when absolute numbers are used, but $r_{\frac{x}{z}z}$ and $r_{\frac{y}{z}z}$ are usually small, and produce little effect on the total correlation $r_{\frac{x}{z}\frac{y}{z}}$, when the partial correlation with z constant is calculated.

* *Journal of the Royal Statistical Society*, LXXIII, 1910, p. 644.

The considerable relative changes produced in $r_{\frac{x}{z}z}$ and $r_{\frac{y}{z}z}$ by the addition of a single observation with a very large population, have comparatively little effect on ${}_zr_{\frac{x}{z}\frac{y}{z}}$. (See Table 3.)

We must now consider whether the forms of the two regression surfaces are likely to vary markedly in the two cases.

We know that the complete interpretation of any coefficient of correlation involves a knowledge of the form of the regression. If the coefficient of correlation between a and b be ·5, and that between c and d be also ·5, it does not follow that the closeness of the relationship is identical in the two cases unless the regression is of the same form in both. This fact suggested the possibility that the difference between ${}_zr_{xy}$ and ${}_zr_{\frac{x}{z}\frac{y}{z}}$ might also depend upon a want of congruence between the forms of their respective regression surfaces. If the three variables are distributed normally we know that variations from the mean of one may be represented effectively in terms of variations from the respective means of the others by an equation of the first degree, that is, the regression may be described as planar. Now assuming that variations in x may be represented in terms of variations of y and z by a first degree equation, does it necessarily follow that variations in $\frac{x}{z}$ may similarly be represented in terms of z and $\frac{y}{z}$? Evidently there is no *prima facie* reason. Some *a priori* considerations do indeed point directly away from any such conclusion. In attempting to ascertain the nature of the distribution of $\frac{x}{y}$ in terms of the constants of the distributions of x and y, assuming that $r_{xy} = 0$ one of us obtained the following expressions for the mean and first four moment coefficients :—*

$$\bar{i} = \left(\frac{\bar{x}}{\bar{y}}\right)\left\{1 + v^2{}_x - \frac{\mu'_3}{\bar{y}^3} + \frac{\mu'_4}{\bar{y}^4}\text{ \&c. }\right\} \tag{8}$$

$$M_2 = \left(\frac{\bar{x}}{\bar{y}}\right)^2\left\{v^2{}_x + v^2{}_y - \frac{2\mu'_3}{\bar{y}^3} + 3v^2{}_x v^2{}_y + \frac{3\mu'_4 - 2\mu_2'^2}{\bar{y}^4}\text{ \&c. }\right\} \tag{9}$$

$$M_3 = \left(\frac{\bar{x}}{\bar{y}}\right)^3\left\{\frac{\mu_3}{\bar{x}^3} - \frac{\mu'_3}{\bar{y}^3} + \frac{6\mu_2\mu'_2}{\bar{x}^2\bar{y}^2} + \frac{3\,(\mu'_4 - \mu_2'^2)}{\bar{y}^4}\text{ \&c. }\right\} \tag{10}$$

$$M_4 = \left(\frac{\bar{x}}{\bar{y}}\right)^4\left\{\frac{\mu_4}{\bar{x}^4} + \frac{\mu'_4}{\bar{y}^4} + \frac{6\mu_2\mu'_2}{\bar{x}^2\,\bar{y}^2}\text{ \&c. }\right\} \tag{11}$$

The order of approximation is not the same in the different terms, but the expressions suffice to show that given symmetry in the original distributions, *i.e.*, $\mu_3 = \mu'_3 = 0$, the distribution of indices will not be symmetrical, under the conditions assumed. These results suggest—they do no more—that were the distributions of absolute values *approximately* normal the indices might not be normal.

* *Biometrika*, vii, 532.

It accordingly seemed desirable to go carefully into the question as to whether the regression surfaces were different in the case of indices and absolute numbers.

We must first obtain a condition for planarity analogous to the accepted test for linearity of regression in the case of two variables. The obvious course to pursue was to follow the general lines of Pearson's memoir on skew correlation.* In the notation of that paper, we have:—

$$\frac{Y_{pz_{p'}}}{\sigma_y} = b_1 + b_2\frac{X_p}{\sigma_x} + b_3\frac{Z_{p'}}{\sigma_2} + b_4\frac{Z_{p'}X_p}{\sigma_x\sigma_z} + b_5\frac{X^2_p}{\sigma^2_x} + b_6\frac{Z^2_{p'}}{\sigma^2_z} \quad (12)$$

Multiplying by $n_{x_pz_{p'}}$ and summing for all arrays,

$$0 = Nb_1 + Nb_4r_{xz} + Nb_5 + Nb_6. \quad \text{or } b_1 = (b_4r_{xz} + b_5 + b_6) \quad (13)$$

Substituting the right-hand side of (13) for b_1, multiplying by $\frac{X_p}{\sigma_x}$ summing and dividing, another constant can be substituted for, and all the constants obtained by the same orderly but wearisome process. The condition for planarity is that $b_4 = b_5 = b_6 = 0$.

The reduction can be better expressed if determinants are used. Dr. E. C. Snow (who has also been working at this problem and has kindly allowed us to see his notes) finds that the vanishing of second order terms involves the vanishing of 3 determinants each of the fifth order and containing fourth moments. He has been able to reduce the condition to the evaluation of a single third order determinant, but each constituent of the latter is itself a determinant of the third order.

It is thus clear that a test for planarity along these lines would involve a considerable amount of arithmetic. Snow, however, points out a more elegant test. Pearson in his classical memoir on Skew Correlation proved that if the regression in the case of two variables be linear $\eta^2 = r^2$ and that in all cases $(\eta^2 - r^2)\sigma^2_y$ is the mean square deviation of the regression curve from a straight line of closest fit.

The former statement can be verified at once.

Thus $$N\sigma_y^2\eta^2 = Sn_x(\bar{y}_{n_x} - \bar{y})^2. \quad (14)$$

If the regression be linear, the right-hand side of (14) is—

$$Sn_x\left(\frac{r\sigma_y}{\sigma_x}\{x - \bar{x}\}\right)^2. = N\sigma^2_yr^2 \quad (15)$$

Snow defines a "solid" η (which we will call H) by—

$$H^2 = \frac{S_{xy}n_{xy}(\bar{z}_{n_x'y'} - \bar{z})^2}{N\sigma^2_z} \quad (16)$$

and finds that for planar regression

$$H^2 = R^2 = 1 - \frac{\Delta}{\Delta_{11}}$$

where $$\Delta = \begin{vmatrix} 1, & r_{zx}, & r_{zy}, \\ r_{xz}, & 1, & r_{xy}, \\ r_{zy}, & r_{xy}, & 1, \end{vmatrix}$$

and $$\Delta_{11} = 1 - r^2_{xy}.$$

* "On the General Theory of Skew Correlation and Non-Linear Regression." *Dravers Co. Research Memoirs*, Dulau and Co., London, 1905.

This may be verified as follows :—

Substitute in (16) for $(\bar{z}_{n_{x'y'}} - \bar{z})$ its value in the case of planar regression, viz. :—

$$\frac{r_{xz} - r_{xy}r_{yx}}{1 - r^2_{xy}} \cdot \frac{\sigma_z}{\sigma_x}(x' - \bar{x}) + \frac{r_{zy} - r_{xz}r_{xy}}{1 - r^2_{xy}} \cdot \frac{\sigma_z}{\sigma_y}(y' - \bar{y}) \qquad (17)$$

Multiply by σ^2_z and sum for all values of $n_{x'y'}$ (the frequency of every pair of values of x and y in the population) and we have :—

$$\mathrm{N}\sigma^2_z\mathrm{H}^2 = \frac{\mathrm{N}\sigma^2_z}{1-r^2_{xy}}\left\{\frac{(r_{zx}-r_{zy}r_{xy})^2 + 2r_{xy}(r_{zx}-r_{zy}r_{xy})(r_{zy}-r_{xz}r_{xy}) + (r_{zy}-r\ \ r_{xy})^2}{1-r^2_{xy}}\right\}$$

$$= \frac{\mathrm{N}\sigma^2_z}{1-r^2_{xy}}(r^2_{xz} + r^2_{yz} - 2r_{xy}r_{xz}r_{yz})$$

$$= \mathrm{N}\sigma^2_z\left(1 - \frac{\Delta}{\Delta_{11}}\right)$$

or $\quad \mathrm{H}^2 = 1 - \frac{\Delta}{\Delta_{11}} = \mathrm{R}^2.$

R is a fairly well-known constant, and has been termed by Yule a coefficient of $(n - 1)$-fold correlation.

Its probable error (calculated by Snow) is $\frac{\cdot 67449}{\sqrt{n}} \cdot \frac{\Delta}{\Delta_{11}}$

The calculation of H is a lengthy process involving a knowledge of every cell in the cube xyz, but we believe that this is the most satisfactory method of testing for planarity of regression.

One further theoretical point arises in this connection. Assuming planarity of regression, it is easy to show that the partial correlation ratio squared, *i.e.*, the square of the average correlation ratio of y on x for all values of z, is simply ${}_zr^2_{xy}$.

What value does this take when the condition of planarity is not fulfilled? Can we find a partial correlation ratio which plays the same part in multiple skew correlation as the ordinary ratio does in the skew correlation of two variables?

We have devoted a good deal of time to this problem, but have failed to obtain a satisfactory result. This is probably due to the inadequacy of our mathematical technic since a statement in *Biometrika* seems to imply that a solution has been obtained and will eventually appear.* However, the results here given are possibly sufficient for the object we have in view. As will be seen later on, various empirical attempts to obtain some function of the single correlation ratios analogous to the coefficients of partial correlation were fruitless.

In order to test the validity of our ideas it was necessary to obtain sufficiently large samples of material to allow of the formation of partial correlation tables. Among the data we were actually working at for other purposes, only a single set approximated to these requirements, viz., 118 English towns of which we knew the populations and also the numbers of deaths in them from cancer and diabetes. The results of analysing this material are communicated below, but it was in any case too sparse to allow of testing the planarity of regression.

* *Biometrika*, viii, 439.

We accordingly collected material *ad hoc*. The process adopted was to go through the report of the Registrar General for 1901, and to take out the first thousand registration subdistricts with populations between 1,000 and 10,000, together with the corresponding numbers of births and deaths. Birth and death rates were computed and the necessary correlation tables drawn up (Tables 4–9). These data were then completely analysed, and the constants deduced appear in Table 10.

We also calculated the skewness of each distribution (Table 12), and the appropriate association constants for each double array corresponding to a tabular population value (Table 15). The coefficients for these arrays are what Pearson and Heron term plural partial correlations.* Lastly we have tested by means of (16), &c., the planarity of regression in two important cases (Table 13). The extreme laboriousness of the arithmetic precluded us from applying this test to each possible regression.

In view of the fact that the thousand subdistricts included one in which, probably owing to the presence of a large hospital, the death rate was very abnormal, we recalculated the principal constants for the 999 which remained after omission of the outlying value. This has had some effect on the planarity test (Table 14), and has also emphasised the difference between the two partial coefficients.

The general impression produced by our results is as follows: The departure from planarity is decidedly more marked in the case of the indices than in that of the absolute values. We should therefore expect that $_{z}r_{xy}$ would be somewhat larger than $_{z}r_{\frac{x}{z}\frac{y}{z}}$ since if, *e.g.*, two variables a and b are as closely associated as c and d, but the regression more nearly linear in the former case, the coefficient of correlation will be greater in that case.† The expectation is realised distinctly in the case of the 999 districts. For the original thousand, however, neither value is significant having regard to its probable error. We think, therefore, that the results are consistent with a belief that the differences between $_{z}r_{xy}$ and $_{z}r_{\frac{x}{z}\frac{y}{z}}$ depend upon differences in the nature of the regression surfaces. It is worthy of note that in this particular instance $_{z}r_{xy}$ and $_{z}r_{\frac{x}{z}\frac{y}{z}}$ do not differ sufficiently for any serious divergence in interpretation to have been likely to result if the material had been

* We have also inserted the corresponding correlation ratios, but these, owing to the small numbers of observations and their scattered distribution, are unreliable. As a warning we also give the theoretical value of the ratio for such samples taken from an uncorrelated population (Pearson, *Biometrika*, viii, 254–6).

† It should, however, be noted that in dealing with short series showing marked variability, although the regression in the case of the indices may be less planar than in the case of the absolute numbers, $_{z}r_{\frac{x}{z}\frac{y}{z}}$ may be greater than $_{z}r_{xy}$, owing to the fact that, in the latter case, certain large values may have had great weight in determining the slope of the regression surface.

used as a basis of some argument (for which purpose it is, of course, entirely unsuitable) by two statisticians, who employed respectively different constants; one ${}_zr_{xy}$, and the other ${}_zr_{\frac{x}{z}\frac{y}{z}}$.

It will be noticed that the material is not nearly so variable as the ordinary series such as are met with in the previous tables. This suggests, a single trial is of course not conclusive, that when the data are numerous and the variation not very considerable, although too great to justify (2)–(5), it is a matter of indifference which coefficient is employed and that indeed no serious risk is run by calculating merely $r_{\frac{x}{z}\frac{y}{z}}$.

We attempted, as mentioned above, to obtain some empirical measure of association which should correspond to the coefficient of partial correlation and be applicable in the case of skew correlation. We cannot simply replace r by η in the ordinary expression for a coefficient of partial correlation and it is hardly correct to say that in skew correlation η plays the same part as r does in normal correlation. Each surface possesses two correlation ratios which may and often do differ considerably. The analogy is rather between η and the corresponding cofficient of regression than between η and r. This suggests that we might replace r not by one of the two correlation ratios but by their geometric mean. As will be seen from the table (Table 16) this artifice does not lead to substantially better agreement. In one case, there is an improvement, in the other the reverse.

We may deal more briefly with the analysis of the 118 towns. As will be seen from Table 17 the total correlations for the absolute values are somewhat more nearly linear than are those for the rates. The data are, however, sparse, and in any case a simple consideration of the total regressions pair and pair does not throw sufficient light upon the nature of the regression of one variable upon the other two. It is, however, worth noting that in no single case is the departure from linearity very marked and that the final agreement between ${}_zr_{xy}$ and ${}_zr_{\frac{x}{z}\frac{y}{z}}$ is quite reasonable.

The practical conclusions to be drawn from this study (which has been arithmetically far more laborious than the reader may be tempted to suppose) seem to be the following.

(1) The differences which are found to occur between ${}_zr_{xy}$ and ${}_zr_{\frac{x}{z}\frac{y}{z}}$ may be attributed to differences in the slope and form of their respective regression surfaces. These differences are due in part to the fact that in calculating ${}_zr_{xy}$ each observation is weighted by its actual size.

(2) In long series of observations where the variation is small the difference is not very marked, and either value may be used.

(3) In such series our experiments suggest that ${}_zr_{xy}$ will be generally slightly greater than ${}_zr_{\frac{x}{z}\frac{y}{z}}$ and the regression surfaces connecting z, y and x will deviate less markedly from planes than those of $\frac{x}{z}$ and $\frac{y}{z}$.

(4) In short series where the variation is large, of the type mostly encountered in the analysis of morbidity and mortality statistics, ${}_{z}r_{xy}$ and ${}_{z}r_{\frac{x}{z}\frac{y}{z}}$ may differ considerably.

(5) In these cases ${}_{z}r_{\frac{x}{z}\frac{y}{z}}$ being less influenced by wide variations in the population totals (which are always highly correlated with the absolute numbers of deaths unless we are dealing with limited outbreaks of contagious disease) is probably the better constant to use.

(6) For rough purposes, $r_{\frac{x}{z}\frac{y}{z}}$ will generally suffice owing to the usually low values of $r_{\frac{x}{z}z}$ and $r_{\frac{y}{z}z}$.

TABLE 1.—*A comparison between the correlation coefficients obtained when absolute numbers and indices are used.*

Nature of the data.	${}_{z}r_{xy}$.*	${}_{z}r_{\frac{x}{z}\frac{y}{z}}$.	$r_{\frac{x}{z}\frac{y}{z}}$.
(1) *Switzerland.*			
(*a*) Cancer and tuberculosis (crude) for 25 cantons	+ ·0697 ± ·1343	− ·1297 ± ·1326	− ·2285 ± ·1279
(*b*) Cancer and diabetes (corrected)† for 25 cantons	− ·0337 ± ·1347	− ·1533 ± ·1317	− ·1756 ± ·1307
(2) *Italy.*			
(*a*) Cancer and diabetes (corrected) for 16 provinces	+ ·1640 ± ·1641	+ ·4425 ± ·1356	+ ·3875 ± ·1433
(*b*) Cancer and diabetes (corrected) for 69 provinces	+ ·1566 ± ·0792	+ ·2151 ± ·0774	+ ·1900 ± ·0783
(3) *England.*			
(*a*) Cancer and diabetes (crude) for 118 English towns (populations, 50,000—375,000)	+ ·3892 ± ·0527	+ ·3566 ± ·0542	+ ·3564 ± ·0542
(*b*) Cancer and diabetes (corrected) for 118 English towns	+ ·1259 ± ·0611	+ ·0438 ± ·0620	+ ·0475 ± ·0619
(*c*) Cancer and diabetes (corrected by Pearson's method)‡ for 118 English towns	+ ·2461 ± ·0583	+ ·0276 ± ·0620	+ ·0285 ± ·0620
(*d*) Cancer and diabetes (crude) for 41 English counties	− ·0265 ± .1053	+ ·6334 ± ·0631	+ ·6635 ± ·0590
(*e*) Cancer and diabetes (corrected by Pearson's method)‡ for 41 English counties	− ·2752 ± ·0974	+ ·4135 ± ·0873	+ ·4272 ± ·0861
(4) *United States of America.*§			
(*a*) Cancer and diabetes (crude) for 40 American cities	+ ·6896 ± ·0559	+ ·3816 ± ·0911	+ ·3847 ± ·0909
(*b*) Cancer and diabetes (corrected) for 40 American cities	+ ·7325 ± ·0494	+ ·6602 ± ·0602	+ ·6817 ± ·0571
(*c*) Cancer and diabetes (corrected) for 33 American cities (omitting all cities of over 500,000 inhabitants)	+ ·8635 ± ·0299	+ ·7415 ± ·0529	+ ·6572 ± ·0667

* Where z = population and x and y deaths from the two diseases.

† Corrected for age-distribution by the ordinary method.

‡ See "On the Correlation of Death-Rates," by Karl Pearson, F.R.S., assisted by Alice Lee, D.Sc., and Ethel M. Elderton, Galton Research Scholar. *Journal of the Royal Statistical Society*, vol. 73, p. 534. The correlations given above are:—${}_{cfz}r_{xy}$, ${}_{cfz}r_{\frac{x}{z}\frac{y}{z}}$ and ${}_{cf}r_{\frac{x}{z}\frac{y}{z}}$, where *cf* = corrective factor.

§ Some of the coefficients of correlation as well as data for the calculation of others were obtained from a paper by G. D. Maynard, F.R.C.S.E., entitled "A Statistical Study in Cancer Death Rates," in *Biometrika*, vol. vii, p. 276.

TABLE 2.

x	y	z
5	7	100
10	14	150
15	16	180
20	18	200
25	21	250
30	30	300

These give :—

$$r_{xy} = ·96306,\quad r_{xz} = ·99124,\quad r_{yz} = ·98435,\quad {}_z r_{xy} = -·54411.$$
$$r_{\frac{x}{z}\frac{y}{z}}\ ·63273,\quad r_{\frac{x}{z}z} = ·88852,\quad r_{\frac{y}{z}z} = ·69366,\quad r_{\frac{x}{z}\frac{y}{z}} = +·04962.$$

adding to the above :—

x	y	z
260	250	3000

we obtain :—

$$r_{xy} = ·99364,\quad r_{xz} = ·99945,\quad r_{yz} = ·99976,\quad {}_z r_{xy} = +·58912.$$
$$r_{\frac{x}{z}\frac{y}{z}} = ·99870,\quad r_{\frac{x}{z}z} = ·99955,\quad r_{\frac{y}{z}z} = ·99898,\quad {}_z r_{\frac{x}{z}\frac{y}{z}} = +·12644.$$

TABLE 3.—*Table showing the effect of the presence of large values upon the coefficient correlation when absolute numbers and indices are used.*

Nature of the data.	Correlations for: Absolute numbers. r_{zy}	r_{xz}	r_{yz}	${}_z r_{xy}$ *	Indices. $r_{\frac{x}{z}\frac{y}{z}}$	$r_{\frac{x}{z}z}$	$r_{\frac{y}{z}z}$	${}_z r_{\frac{x}{z}\frac{y}{z}}$
Correlation between deaths from cancer and diabetes (corrected) for 40 American cities :—†								
(1) Original values...............	+ ·9712 ± ·0061	+ ·9898 ± ·0022	+ ·9474 ± ·0109	+ ·7325 ± ·0494	+ ·6802 ± ·0573	+ ·3511 ± ·0935	+ ·2263 ± ·1012	+ ·6587 ± ·0604
(2) With the addition of an imaginary observation with z=3,740,000, and with mean cancer and diabetes death-rates	+ ·9827 ± ·0036	+ ·9913 ± ·0018	+ ·9631 ± ·0076	+ ·7883 ± ·0399	+ ·6802 ± ·0573	+ ·2284 ± ·0998	+ ·1476 ± ·1030	+ ·6714 ± ·0579
(3) With the addition of an imaginary observation with z=3,740,000, but with highest cancer death-rate and lowest diabetes death-rate found among the 40 original cities.........	+ ·7846 ± ·0405	+ ·9833 ± ·0035	+ ·8539 ± ·0285	− ·5808 ± ·0698	+ ·4639 ± ·0837	+ ·5652 ± ·0717	− ·0373 ± ·1052	+ ·5884 ± ·0689
Correlation between deaths from cancer and diabetes (crude) for English counties :—								
(1) 32 rural and semi-rural counties	+ ·9662 ± ·0079	+ ·9517 ± ·0112	+ ·8973 ± ·0232	+ ·8286 ± ·0374	+ ·6600 ± ·0673	− ·6151 ± ·0741	− ·5617 ± ·0816	+ ·4821 ± ·0915
(2) With the addition of nine urban counties of which three have very large populations‡	+ ·9741 ± ·0054	+ ·9852 ± ·0031	+ ·9895 ± ·0022	− ·0265 ± ·1053	+ ·6635 ± ·0590	− ·2655 ± ·0979	− ·3197 ± ·0946	+ ·6334 ± ·0631

* Where z = population, x = number of deaths from cancer, and y = number of deaths from diabetes.

† See " A Statistical Study in Cancer Death-rates," by G. D. Maynard, F.R.C.S.E., Pretoria. *Biometrika*, vol. vii, p. 276.

‡ See note ‡ on next page.

TABLE 3 *Contd.—Showing effect of presence of large values upon coefficient of correlation*

Nature of the data.	Means and standard deviations. Absolute numbers.				Indices.				Mean population.	S.D. population
	Mean x.	σ_x	Mean y.	σ_y	Mean $\frac{x}{z}$*	$\sigma_{\frac{x}{z}}$	Mean $\frac{y}{z}$*	$\sigma_{\frac{y}{z}}$		
Correlation between deaths from cancer and diabetes (corrected) for 40 American cities :—†										
(1) Original values	294·29	390·36	42·857	63·513	732·63	128·67	105·85	32·152	374,000	449,24
(2) With the addition of an imaginary observation with $z = 3,740,000$, and with mean cancer and diabetes death-rates	353·95	539·43	51·468	83·074	732·63	127·09	105·85	31·757	456,098	683,00
(3) With the addition of an imaginary observation with $z = 3,740,000$, but with highest cancer death-rate and lowest diabetes death-rate found among the 40 original cities	395·12	745·16	46·829	67·576	743·63	144·92	104·61	32·712	456,098	683,00
Correlation between deaths from cancer and diabetes (crude) for English counties :—										
(1) 32 rural and semi-rural counties	192·77	111·72	26·229	14·403	898·91	132·70	123·28	22·456	228,488	154,47
(2) With the addition of the nine urban counties of which three have very large populations‡	313·97	406·61	41·984	50·711	870·45	138·94	118·98	21·811	380,844	475,03

* Death-rate per 1,000,000 living.

† See "A Statistical Study in Cancer Death-rates," by G. D. Maynard, F.R.C.S.E., Pretoria. *Biometrika* vol. vii, p. 276.

‡

	Population. Males.	Cancer death-rate per 1,000,000.	Diabetes death-rate per 1,000,000.
London	2,131,900	1,033	107
Lancashire	2,242,500	727	107
West Riding of Yorkshire	1,427,100	748	120

TABLE 4.—*Correlation table: Births and population.* (1,000 *English registration sub-districts.*)

Population Groups.	Births.																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100	100–110	110–120	120–130	130–140	140–150	150–160	160–170	170–180	180–190	190–200	200–210
1,000– 1,500 ..	2·5	10	11·5	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
1,500– 2,000 ..	..	4·5	11·5	19·5	4	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..
2,000– 2,500 ..	..	..	4	25·5	23·5	11·5	3	..	..	..	..	..	..	..	..	..	..	..	..	..
2,500– 3,000 ..	..	..	..	2·5	16·5	32·5	14·5	7·5	2	1	..	..	..	..	..	..	..	..	..	..
3,000– 3,500 ..	..	..	..	..	5·5	19·5	25·5	25·5	15	3·5	1	..	..	..	..	..	..	..	..	..
3,500– 4,000 ..	..	..	..	..	..	7	20·5	20·5	22	13	3	3	..	..	..	..	..	..	..	..
4,000– 4,500 ..	..	..	..	..	..	1	1	12	24	15	13	3	..	..	..	..	..	..	..	..
4,500– 5,000 ..	..	..	..	..	..	..	..	2	10·5	9·5	17	8·5	3·5	3	1	..	..	..	..	..
5,000– 5,500 ..	..	..	..	..	..	..	..	1	2·5	6·5	9·5	22·5	11	7	5	3	..	..	..	..
5,500– 6,000 ..	..	..	..	..	..	..	..	..	1	3	4	11	18·5	10	6·5	7	..	2	..	1
6,000– 6,500 ..	..	..	..	1	..	..	..	..	..	2·5	2	1·5	12	11	8	7·5	4	2·5	2·5	1
6,500– 7,000 ..	..	..	..	..	..	..	..	..	..	·5	1	1·5	2	6	6·5	11·5	5	4·5	3	3
7,000– 7,500 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	4	3	6	7	7	10·5	3·5
7,500– 8,000 ..	..	..	..	..	..	..	..	..	..	..	1	1	..	..	..	3·5	5·5	10	8·5	8
8,000– 8,500 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	·5	·5	4·5	2·5	5	4	6·5
8,500– 9,000 ..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	1	..	2·5	4	4·5	6·5
9,000– 9,500 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1	3	3·5
9,500–10,000 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	2	1·5
Total ..	2·5	14·5	27	49·5	49·5	74·5	64·5	68·5	77	54·5	51·5	52	47	41·5	31·5	43	28·5	36	38	34·5

TABLE 4.—*Correlation table: Births and population. (1,000 English registration sub-districts.)—Contd.*

Population Groups.	Births. 21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	Total Frequency.
	210–220	220–230	230–240	240–250	250–260	260–270	270–280	280–290	290–300	300–310	310–320	320–330	330–340	340–350	350–360	360–370	
1,000– 1,500 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	25
1,500– 2,000 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	41·5
2,000– 2,500 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	67·5
2,500– 3,000 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	76·5
3,000– 3,500 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	95·5
3,500– 4,000 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	89
4,000– 4,500 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	69
4,500– 5,000 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	55
5,000– 5,500 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	68
5,500– 6,000 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	64
6,000– 6,500 ..	2	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	58·5
6,500– 7,000 ..	1	1	..	..	..	..	1	..	..	..	..	..	..	..	..	..	47·5
7,000– 7,500 ..	2·5	3·5	·5	·5	..	..	..	..	..	..	..	..	..	1	..	..	49
7,500– 8,000 ..	3·5	5	1	3	1	1	..	..	..	..	..	..	..	..	..	..	52
8,000– 8,500 ..	6·5	6·5	1·5	2	2	..	2	1	..	..	..	..	..	..	..	..	45
8,500– 9,000 ..	6	4	1·5	3·5	1·5	1	..	..	2	1	..	..	..	..	..	1	41
9,000– 9,500 ..	2·5	5·5	7	3	1	1·5	1	1	..	..	..	..	..	..	..	..	31
9,500–10,000 ..	2·5	2	2·5	4·5	2	3·5	1·5	1	..	..	..	1	..	..	..	..	25
Total ..	26·5	27·5	14	17·5	7·5	7	5·5	3	2	1	—	1	—	1	—	1	1,000

TABLE 5.—*Correlation table: Deaths and population.* (1,000 *English registration sub-districts.*)

Population Groups.		Deaths.														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100	100–110	110–120	120–130	130–140	140–150
1,000– 1,500	..	1·5	20	3·5	..	..	..	..	..	..	..	..	..	..	..	..
1,500– 2,000	..	..	8·5	28·5	4·5	..	..	..	..	..	..	..	..	..	..	..
2,000– 2,500	..	..	5	37	19	6	·5	..	..	..	..	..	..	..	..	..
2,500– 3,000	..	..	·5	11	32·5	24·5	8	..	..	..	..	..	..	..	..	..
3,000– 3,500	..	..	..	2·5	26·5	44·5	19	3	..	..	..	..	..	..	..	..
3,500– 4,000	..	..	..	2·5	7·5	30·5	29·5	11	6	1	1	..	..	..	..	..
4,000– 4,500	..	..	..	..	1	14·5	17·5	24·5	9	·5	1	..	..	..	..	..
4,500– 5,000	..	..	..	..	..	5·5	12·5	22	9	2·5	2·5	..	..	..	..	1
5,000– 5,500	..	..	..	..	..	1·5	9	17	21	8·5	6	2·5	·5	..	..	1
5,500– 6,000	..	..	..	..	..	·5	1·5	7	23·5	9	16·5	3	..	..	1	..
6,000– 6,500	..	..	..	..	..	..	2	4	5	18·5	13·5	10·5	1·5	·5	1	1
6,500– 7,000	..	..	..	..	..	..	..	1	7	4·5	13·5	9	7·5	2	1	..
7,000– 7,500	..	..	..	..	..	..	..	1	3	7	12	11·5	5·5	4	1·5	1·5
7,500– 8,000	..	..	..	..	..	..	..	..	2·5	2	8	11	14	7·5	2·5	·5
8,000– 8,500	..	..	..	..	..	..	..	..	1	..	3	10	12·5	7·5	5	1
8,500– 9,000	..	..	..	..	..	..	..	..	..	1	4	2·5	4	10	4·5	6·5
9,000– 9,500	..	..	..	..	..	..	..	..	..	1	2	5	1	7	8·5	1·5
9,500–10,000	..	..	..	..	..	..	..	..	..	..	1	1	2·5	3·5	2	8
Total	..	1·5	34	85	91	127·5	99·5	90·5	87	55·5	84	66	49	42	27	22

TABLE 5.—*Correlation table: Deaths and population.* (1,000 *English registration sub-districts.*)—*Contd.*

Population Groups.	Deaths.														Total Frequency.
	16	17	18	19	20	21	22–29	30	31–33	34	35–38	39	40–74	75	
	150–160	160–170	170–180	180–190	190–200	200–210	210–290	290–300	300–330	330–340	340–380	380–390	390–740	740–750	
1,000– 1,500 ..	..	..	..	..	..	..	..	..	..	..	..		..	..	25
1,500– 2,000 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	41·5
2,000– 2,500 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	67·5
2,500– 3,000 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	76·5
3,000– 3,500 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	95·5
3,500– 4,000 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	89
4,000– 4,500 ..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	69
4,500– 5,000 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	55
5,000– 5,500 ..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	68
5,500– 6,000 ..	..	1	..	..	1	..	..	..	..	..	..	..	..	..	64
6,000– 6,500 ..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	58·5
6·500– 7,000 ..	..	..	..	·5	·5	1	..	..	..	..	..	..	..	..	47·5
7,000– 7,500 ..	..	1	·5	·5	..	..	..	..	..	..	..	..	..	..	49
7,500– 8,000 ..	..	2	..	..	1	..	..	..	..	..	..	..	..	1	52
8,000– 8,500 ..	1	1	1	..	..	1	..	..	..	..	..	1	..	..	45
8,500– 9,000 ..	2·5	2	1	1	1	..	..	..	..	1	..	..	..	..	41
9,000– 9,500 ..	3	..	1	1	..	..	..	..	..	..	..	..	..	..	31
9,500–10,000 ..	1	2	2	·5	·5	..	..	1	..	..	..	..	..	..	25
Total ..	9·5	9	6·5	3·5	4	2	—	1	—	1	—	1	—	1	1,000

Table 6.—*Correlation table: Births and deaths.* (1,000 *English registration sub-districts.*)

Births.	Deaths. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100	100–110	110–120	120–130	130–140	140–150
10– 20..	..	2·5	..	..	..	..	..	..	..	..	..	..	..	..	..
20– 30..	..	8	6	·5	..	..	..	..	..	..	..	..	..	..	..
30– 40..	1·5	14·5	8·5	·5	1·5	·5	..	..	..	..	..	..	..	..	..
40– 50..	..	5·75	27·25	12	3·5	..	..	..	..	..	..	..	..	..	..
50– 60..	..	1·75	21·75	16	9	1	..	..	..	..	..	..	..	..	..
60– 70..	..	1·5	16	23	20	9	3	1	..	1	..	..	..	..	..
70– 80..	..	..	1·5	18·75	25·25	13	3	3	..	..	..	..	..	..	..
80– 90..	..	..	3·5	14·25	26·75	12·5	5·5	4	·5	·5	..	..	..	..	..
90–100..	..	..	·5	4	22	21	17	6	1	3·5	..	..	..	..	1
100–110..	..	..	..	2	13·5	16	12	4	2	4	1	..	..	..	..
110–120..	..	..	..	..	3	12·75	17·75	7·5	3·5	5	..	..	..	..	1
120–130..	..	..	..	..	2·5	6·25	14·25	16·5	7·5	4	..	..	..	1	..
130–140..	..	..	..	..	·5	2·5	9	14·5	7·5	9	3	..	..	..	..
140–150..	..	..	..	..	..	4	2·5	9·5	7	8	7·5	1	..	1	..
150–160..	..	..	..	..	..	..	3·5	6·5	6·5	3	2·75	3·25	1	..	1
160–170..	..	..	..	..	..	1	2	6	3	11·5	12·25	3·25	3	..	..
170–180..	..	..	..	..	..	..	..	3	2·5	6·25	5·25	6·5	2	..	..
180–190..	..	..	..	..	..	..	..	2	4·5	9·75	5·25	6	6·25	2·25	..
190–200..	..	..	..	..	..	..	1	·5	6	5	4·25	8·25	4·75	3	5·25
200–210..	..	..	..	..	..	..	..	2	2	6	7·75	7·25	3	2·25	1·75
210–220..	..	..	..	..	..	..	..	..	1	2·5	6·5	1	7	2·5	2·5
220–230..	..	..	..	..	..	..	..	1	..	2	3·5	4	4·5	5·5	2·5
230–240..	..	..	..	..	..	..	..	..	..	1	5	·5	1	2	1
240–250..	..	..	..	..	..	..	..	..	1	1	1·5	3·5	4	1	4·5
250–260..	..	..	..	..	..	..	..	..	..	..	·5	1·5	2	..	·5
260–270..	..	..	..	..	..	..	..	..	..	1	..	2	..	1·5	1
270–280..	..	..	..	..	..	..	..	..	..	..	..	1	..	2·5	..
280–290..	..	..	..	..	..	..	..	..	..	..	..	..	1	1	..
290–300..	..	..	..	..	..	..	..	..	..	..	..	..	·5	1·5	..
300–310..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
310–320..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
320–330..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
330–340..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
340–350..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..
350–360..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
360–370..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..
Total ..	1·5	34	85	91	127·5	99·5	90·5	87	55·5	84	66	49	42	27	22

TABLE 6.—*Correlation table: Births and deaths.* (1,000 *English registration sub-districts.*)—*Contd.*

Births.	Deaths.														Total Frequency.
	16	17	18	19	20	21	22–29	30	31–33	34	35–38	39	40–74	75	
	150–160	160–170	170–180	180–190	190–200	200–210	210–290	290–300	300–330	330–340	340–380	380–390	390–740	740–750	
10– 20	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2·5
20– 30	..	..	..	..	..	..	..	..	..	..	..	..	..	..	14·5
30– 40	..	..	..	..	..	..	..	..	..	..	..	..	..	..	27
40– 50	..	..	1	..	..	..	..	..	..	..	..	..	..	..	49·5
50– 60	..	..	..	..	..	..	..	..	..	..	..	..	..	..	49·5
60– 70	..	..	..	..	..	..	..	..	..	..	..	..	..	..	74·5
70– 80	..	..	..	..	..	..	..	..	..	..	..	..	..	..	64·5
80– 90	1	..	..	..	..	..	..	..	..	..	..	..	..	..	68·5
90–100	1	..	..	..	..	..	..	..	..	..	..	..	..	..	77
100–110	..	..	..	..	..	..	..	..	..	..	..	..	..	..	54·5
110–120	..	..	..	..	..	..	..	..	..	..	..	..	..	1	51·5
120–130	..	..	..	..	..	..	..	..	..	..	..	..	..	..	52
130–140	..	1	..	..	..	..	..	..	..	..	..	..	..	..	47
140–150	..	..	·5	·5	..	..	..	..	..	..	..	..	..	..	41·5
150–160	..	1	..	·5	1·5	..	..	..	..	1	..	..	..	..	31·5
160–170	..	..	..	..	..	1	..	..	..	..	..	..	..	..	43
170–180	1	2	..	..	..	..	..	..	..	..	..	..	..	..	28·5
180–190	..	..	..	..	..	..	..	..	..	..	..	..	..	..	36
190–200	..	..	..	..	..	..	..	..	..	..	..	..	..	..	38
200–210	..	·5	1	1	..	..	..	..	..	..	..	..	..	..	34·5
210–220	1	2·5	..	..	..	..	..	..	..	..	..	..	..	..	26·5
220–230	..	1·5	1	..	1	1	..	..	..	..	..	..	..	..	27·5
230–240	2	·5	..	·5	·5	..	..	..	..	..	..	..	..	..	14
240–250	..	..	..	·5	·5	..	..	..	..	..	..	..	..	..	17·5
250–260	·5	..	1·5	·5	5	..	..	..	..	..	..	..	..	..	7·5
260–270	1	..	·5	..	..	..	..	..	..	..	..	..	..	..	7
270–280	..	..	1	..	..	..	..	..	..	..	..	1	..	..	5·5
280–290	1	..	..	..	..	..	..	..	..	..	..	..	..	..	3
290–300	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2
300–310	1	..	..	..	..	..	..	..	..	..	..	..	..	..	1
310–320	..	..	..	..	..	..	..	..	..	..	..	..	..	..	—
320–330	..	..	..	..	..	..	..	1	..	..	..	..	..	..	1
330–340	..	..	..	..	..	..	..	..	..	..	..	..	..	..	—
340–350	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
350–360	..	..	..	..	..	..	..	..	..	..	..	..	..	..	—
360–370	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
Total ..	9·5	9	6·5	3·5	4	2	—	1	—	1	—	1	—	1	1,000

TABLE 7.—*Correlation table: Birth rates and population.* (1,000 *English registration sub-districts.*)

Population Groups.	Birth Rates. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
	70–80	80–90	90–100	100–110	110–120	120–130	130–140	140–150	150–160	160–170	170–180	180–190	190–200	200–210	210–220	220–230	230–240	240–250	250–260	260–270	270–280	280–290	290–300
1,000– 1,500 ..	..	..	..	..	..	..	..	..	1	2	1	..	2	1	2	3	2·5	1·5	2	..	1·5	1·5	1
1,500– 2,000 ..	..	..	..	..	..	..	..	1	..	1	..	5	2·5	1·5	2	4	6	6	4	2	1·25	·25	1
2,000– 2,500 ..	..	..	..	..	..	..	..	..	2	·5	2·5	4	6	6	6·5	5·5	6	6	6	3	2·25	3·75	1·5
2,500– 3,000 ..	..	..	..	..	..	..	..	..	..	1	2	1	3	8·5	10	4·5	7·5	11·75	4·25	7·5	6·5	2	2
3,000– 3,500 ..	..	..	..	..	..	..	..	..	..	..	1	4	4	5·5	7·5	12	12	10·25	9·25	8·5	8	4·5	5
3,500– 4,000 ..	..	..	..	..	..	..	..	..	..	2	3	3	6	9	9	10	8	8·5	5·5	11	4	2	3
4,000– 4,500 ..	..	..	..	..	..	..	..	..	..	1	..	2	3	5	8	13·5	8	5·5	8	8	4	..	3
4,500– 5,000 ..	..	..	..	..	..	..	..	..	..	..	1	2·5	4·5	3	5·5	5·5	5·5	11	5·5	3	2·5	1·5	1
5,000– 5,500 ..	..	..	..	..	..	..	..	..	..	1	..	..	4	4	6·5	7	9	11·5	7·5	4·5	2·5	5·5	2
5,500– 6,000 ..	..	..	..	..	..	..	..	..	..	1	2	1	1	2	4·5	10	11·5	11	2	7	4	1	3
6,000– 6,500 ..	1	..	..	..	..	..	..	..	..	..	2	2	1	3	5	8·5	7·5	4	6	3·5	5	·5	1·5
6,500– 7,000 ..	..	..	..	..	..	..	..	..	1	..	..	1	2	1·5	2·5	7	4	8·5	3·5	4·5	2	3	1
7,000– 7,500 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	3·5	·5	5·5	7	4	8·5	5	5·5	1·5	1·5
7,500– 8,000 ..	..	..	..	..	..	..	..	1	1	..	..	..	..	..	3·5	5·5	5	7·5	7·5	7	4·5	·5	3
8,000– 8,500 ..	..	..	..	..	..	..	..	..	..	..	..	1	1	4	3	4	4	4	6·5	4·5	4·5	·5	3
8,500– 9,000 ..	1	..	..	..	..	..	..	..	..	..	1	..	1	2	6	2	6·5	5·5	4	2	2	2	1
9,000– 9,500 ..	..	..	..	..	..	..	..	..	..	..	..	..	2	..	5	3	1·5	6·5	7	2	..	2	1
9,500–10,000 ..	..	..	..	..	..	..	..	..	..	..	1	..	·5	1·5	3	2	1	4·5	2·5	4·5	2·5	..	..
Total ..	2	—	—	—	—	—	—	2	5	9·5	16·5	26·5	43·5	61	90	112·5	112·5	127·5	99·5	87·5	62·5	32·0	34·5

TABLE 7.—*Correlation table: Birth rates and population.* (1,000 *English registration sub-distri ts*).—*Contd.*

Population Groups.	Birth Rates.																				Total Frequency.
	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	
	300–310	310–320	320–330	330–340	340–350	350–360	360–370	370–380	380–390	390–400	400–410	410–420	420–430	430–440	440–450	450–460	460–470	470–480	480–490	490–500	
1,000– 1,500 ..	1	1	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	25
1,500– 2,000 ..	1	1	1	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	41·5
2,000– 2,500 ..	2	3	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	67·5
2,500– 3,000 ..	2·5	·5	1	..	·5	·5	..	..	..	..	..	..	..	..	..	..	..	..	..	..	76·5
3,000– 3,500 ..	2	1	..	·5	·5	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	95·5
3,500– 4,000 ..	1·5	3·5	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	89
4,000– 4,500 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	69
4,500– 5,000 ..	1·5	1·5	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	55
5,000– 5,500 ..	·5	2·5	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	68
5,500– 6,000 ..	..	1	1	..	·5	·5	..	..	..	..	..	..	..	..	..	..	..	..	..	..	64
6,000– 6,500 ..	3	1	..	2	1	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	58 5
6,500– 7,000 ..	2·5	·5	2	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	47·5
7,000– 7,500 ..	2	2·5	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	49
7,500– 8,000 ..	..	2	2	1	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	52
8,000– 8,500 ..	1	1	1	..	1	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	45
8,500– 9,000 ..	1	..	..	1	2	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	41
9,000– 9,500 ..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	31
9,500–10,000 ..	1	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	25
Total ..	22·5	23	10	5·5	7·5	3	—	—	—	2	—	1	—	—	—	—	—	—	—	1	1,000

TABLE 8.—*Correlation table : Death rates and population.* (1,000 *English registration sub-districts.*)

Population Groups.	Death Rates.																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	60–70	70–80	80–90	90–100	100–110	110–120	120–130	130–140	140–150	150–160	160–170	170–180	180–190	190–200	200–210	210–220	220–230	230–240	240–250
1,000– 1,500..	..	2	2·5	2·5	2	2	1	2	6	3	..	..	1·5	·5	..	..	..	..	..
1,500– 2,000..	..	2	..	2	2·5	5	6	6	5	4	3	1	3	2	..	..	..	..	..
2,000– 2,500..	1·5	·5	2·5	3·5	8	7·5	12·5	12	5·5	1	5	2	1·5	2·5	..	1	1	..	..
2,500– 3,000..	..	1	1·5	3·5	3	6·5	11·5	11	9·75	12·25	7·5	3	1	3	1	1	..	..	..
3,000– 3,500..	..	2	..	4·5	5·5	14·5	13	18·5	11·25	9·25	7·5	6·5	2	..	..	1	..	..	..
3,500– 4,000..	1	1	2	4·5	2·5	13·5	15	14	12	7·5	3	4·5	2·5	3	1	..	1	..	..
4,000– 4,500..	..	..	1	1	6	7·5	10·5	5·5	11	10·5	6·5	4·5	3	..	..	1	..	..	..
4,500– 5,000..	..	..	1	1	4	10	7	9·5	6	8·5	2·5	1·5	2	..	1	..	..	..	..
5,000– 5,500..	..	..	1	2	3	6	9·5	13·5	9	9	5	3	2	..	3	..	..	..	..
5,500– 6,000..	..	..	·5	1·5	2	3·5	9	16	6·5	7·5	9	5·5	..	..	..	..	..	..	1
6,000– 6,500..	..	..	2	..	4	..	7·5	13·5	9	7·5	6·5	4	·5	1	..	..	1	1	..
6,500– 7,000..	..	..	..	1	1·5	7·5	2	4·5	7	9	9	1	1·5	·5	..	1	..	..	..
7,000– 7,500..	..	..	..	1	3	5	6	10	7	6·5	1·5	3	3	1	..	..	1	..	..
7,500– 8,000..	..	..	..	·5	3·5	3	5	8	15·5	4·5	7	..	1	..	1	1	..	..	..
8,000– 8,500..	..	..	..	1	·5	1·5	6·5	9	12	6·5	2·5	·5	1	..	1	1	..	..	..
8,500– 9,000..	..	..	..	1	3·5	1·5	2·5	6·5	7	5	3·5	3·5	2	2	1	1	..	..	..
9,000– 9,500..	..	..	..	1	3	4	2	6·5	6	3·5	3	..	1	..	1	..	..	..	..
9,500–10,000..	..	..	..	1	1	1	3	4	7·5	1·5	1	2	1	1	..	..	..	..	..
Total ..	2·5	8·5	14	32·5	58·5	99·5	129·5	170	153	116·5	83	45·5	29·5	16·5	10	8	4	1	1

TABLE 8.—*Correlation table: Death rates and population.* (1,000 *English registration sub-districts*).—*Contd.*

Population Groups.	Death Rates.																	Total Frequency.
	20	21	22	23	24	25	26–27	28	29	30	31–32	33	34	35–41	42	43–88	89	
	250–260	260–270	270–280	280–290	290–300	300–310	310–330	330–340	340–350	350–360	360–380	380–390	390–400	400–470	470–480	480–940	940–950	
1,000– 1,500..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	25
1,500– 2,000..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	41·5
2,000– 2,500..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	67·5
2,500– 3,000..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	76·5
3,000– 3,500..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	95·5
3,500– 4,000..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	89
4,000– 4,500..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	..	69
4,500– 5,000..	..	..	..	·5	·5	..	..	..	..	..	..	..	..	..	..	..	..	55
5,000– 5,500..	..	..	1	1	..	..	..	..	..	..	..	..	..	..	..	..	..	68
5,500– 6,000..	..	..	..	1	..	..	..	1	..	..	..	..	..	..	..	..	..	64
6,000– 6,500..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	58·5
6,500– 7,000..	..	..	1	..	1	..	..	..	..	..	..	..	..	..	..	..	..	47·5
7,000– 7,500..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	49
7,500– 8,000..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	52
8,000– 8,500..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	45
8,500– 9,000..	..	..	..	..	..	..	..	..	..	..	..	·5	·5	..	..	..	..	41
9,000– 9,500..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	31
9,500–10,000..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	25
Total ..	3	1	2	2·5	2·5	1	—	1	—	1	—	·5	·5	—	1	—	1	1,000

TABLE 9.—*Correlation table: Birth rates and death rates.* (1,000 *English registration sub-districts.*)

Birth Rates.	Death Rates.																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	60–70	70–80	80–90	90–100	100–110	110–120	120–130	130–140	140–150	150–160	160–170	170–180	180–190	190–200	200–210	210–220	220–230	230–240	240–250	250–260
70– 80	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
80–140	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
140–150	..	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..
150–160	..	..	..	·5	·5	..	..	·5	·5	..	..	..	1	1	..	..	1	..	..	..
160–170	..	..	·5	·5	..	2	2	·5	1	1	..	..	..	1	..	..	..	..	..	..
170–180	..	..	..	..	1	1	1	4	1·5	3	1	3	..	..	..	..	..	..	..	..
180–190	..	2	2·5	2·5	3·5	2·5	2·5	3	4·5	1·5	..	..	1	1	..	..	..	..	..	..
190–200	..	..	..	2·5	2	2	9	6·5	6·5	5·5	2·5	2	1	1	2	..	..	..	..	..
200–210	..	..	1·5	6	6	6	11·5	7·5	4·5	2·5	6	1	3·5	1	..	2·5	..	..	..	1·5
210–220	..	1	2	2·5	4·75	12·75	15·25	10·75	13·5	12	6·5	3·5	·5	2	..	1·5	..	..	..	·5
220–230	..	1	1·5	3	5·75	13·75	15·75	16·75	21·25	12·25	6·25	5·75	2·5	1	1	1	1	..	1	..
230–240	2	..	2	3·5	8	10·5	11·75	24·5	14·5	15·25	9·25	4·25	3	2	..	..	1	..	..	..
240–250	·25	1·25	1	2·5	7	14	18	21·25	17·5	12·25	14·5	12·5	2	2·5	1	..	..	..	..	..
250–260	·25	2·25	..	4	6·5	14	10·75	14·75	19	8·5	12·5	2·5	2	1	·5	..	..	1	..	..
260–270	..	1	1	2	3·5	7	11·75	19·5	16	10·75	3·75	·25	4·5	2	2·5	..	..	..	..	..
270–280	..	..	..	3	5·5	4·5	10·75	11	8·75	6·5	5·25	3·25	·5	..	2	·5	1	..	..	..
280–290	..	..	1	..	·5	4	2·5	4	5·5	5	4	..	3	..	1	1·5	..	..	..	..
290–300	..	..	..	..	1	1	3	10	6·5	4·5	3·5	3	1	..	..	..	..	..	..	1
300–310	..	..	..	..	2	1·5	..	3·5	5	4·5	2	2	1	..	..	1	..	..	..	..
310–320	..	..	..	..	..	2·5	2	5·5	4·5	4	2	1·5	..	1	..	..	..	..	..	..
320–330	..	..	1	..	..	..	..	2	·5	4·5	..	..	..	..	..	..	..	..	..	..
330–340	..	..	..	..	..	..	..	2	1	1	1·5	..	..	..	..	..	..	..	..	..
340–350	..	..	..	..	..	·25	1·75	·5	..	1·5	1·5	1	..	..	..	..	..	..	..	..
350–360	..	..	..	..	..	·25	·25	..	..	·5	..	..	2	..	..	..	..	..	..	..
360–370	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
370–380	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
380–390	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
390–400	..	..	..	..	..	..	..	1	..	..	1	..	..	..	..	..	..	..	..	..
400–410	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
410–420	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..
420–490	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
490–500	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	..
Total	2·5	8·5	14	32·5	58·5	99·5	129·5	170	153	116·5	83	45·5	29·5	16·5	10	8	4	1	1	3

TABLE 9.—*Correlation table: Birth rates and death rates.* (1,000 *English registration sub-districts.*)—*Contd.*

Birth Rates.	Death Rates.																								Total Frequency.
	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43-88	89	90	
	260–270	270–280	280–290	290–300	300–310	310–320	320–330	330–340	340–350	350–360	360–370	370–380	380–390	390–400	400–410	410–420	420–430	430–440	440–450	450–460	460–470	470–480	480–940	940–950	950–960	
70– 80 ..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2
80–140 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	—
140–150 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	2
150–160 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	5
160–170 ..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	9·5
170–180 ..	..	..	..	..	..	..	..	..	..	..	..	..	·5	·5	..	..	..	..	..	..	..	..	..	..	..	16·5
180–190 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	26·5
190–200 ..	..	..	·5	·5	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	43·5
200–210 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	61
210–220 ..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	90
220–230 ..	..	1	..	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	112·5
230–240 ..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	112·5
240–250 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	127·5
250–260 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	99·5
260–270 ..	1	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	87·5
270–280 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	62·5
280–290 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	32
290–300 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	34·5
300–310 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	22·5
310–320 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	23
320–330 ..	..	..	..	1	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	10
330–340 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	5·5
340–350 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	7·5
350–360 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	3
360–370 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	—
370–380 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	—
380–390 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	—
390–400 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2
400–410 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	—
410–420 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
420–490 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	—
490–500 ..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
Total ..	1	2	2·5	2·5	1	—	—	1	—	1	—	—	·5	·5	—	—	—	—	—	—	—	1	—	1	—	1,000

TABLE 10.—1,000 *English registration sub-districts.**

Rates—

	Means and standard deviations.
Mean birth-rate	24 ·3495 per 1,000 popn.
S.D. ,, ...	3 ·8539
Mean death-rate...........................	14 ·3360 per 1,000 popn.
S.D. ,, ...	4 ·2851
Mean population...........................	5094 ·7500
S.D. ,, ...	2290 ·5834

Correlation—

	r.	η.	Geometric mean.	Linearity test.†
B.R. and population	·1375 ± ·0209	·1871 ·2281	·2066	2 ·974 4 ·266
D.R. and population	·1478 ± ·0209	·1862 ·2514	·2164	2 ·654 4 ·768
B.R. and D.R.	– ·0152 ± ·0213	·4581 ·2576	·3435	10 ·733 6 ·028

Partial correlation—

B.R. and popn. : D.R. const.	·1413 ± ·0209
D.R. and popn. : D.R. const.	·1514 ± ·0209
B.R. and D.R. : popn. const.	– ·0363 ± ·0213

Absolute numbers—

	Means		Standard deviations
Mean births	125 ·255	S.D.	62 ·6625
Mean deaths	74 ·485	S.D.	46 ·1691
Mean population	5094 ·7500	S.D.	2290 ·5834

Correlation—

	r.	η.	Geometric mean.	Linearity test.†
Births and popn.	·9345 ± ·0027	·9358 ·9501	·9429	1 ·154 4 ·469
Deaths and popn.	·7822 ± ·0083	·7855 ·9241	·8520	1 ·692 11 ·534
Births and deaths	·7333 ± ·0097	·8871 ·7527	·8171	11 ·538 3 ·401
Partial correlation—				
Births and popn.: deaths const.	·8521 ± ·0058			
Deaths and popn. : births const.	·4003 ± ·0179			
Births and deaths : popn. const.	·0106 ± ·0213			

* The above Table includes one sub-district with death-rate 94 ·3 per 1,000 the corresponding birth-rate being 14 ·6 per 1,000. In Table 11 will be found the correlations omitting this extreme case.

† See Table 15.

TABLE 11.—999 *English registration sub-districts.*

Rates—

	Means and standard deviations.
Mean birth-rate	24·3594 per 1,000 popn.
S.D. „	3·8432
Mean death-rate	14·2558 per 1,000 popn.
S.D. „	3·4556
Mean population	5092·0921
S.D. „	2290·1878

Correlation—

	r.	η.	Geometric mean.	Linearity test.*
B.R. and population	·1411 ± ·0209	{·2330 ·1885}	·2095	{4·347 2·930
D.R. and population	·1566 ± ·0208	{·1988 ·2489}	·2225	{2·871 4·535
B.R. and D.R.	·0785 ± ·0212	{·2425 ·2454}	·2439	{5·379 5·450

Partial correlation—

	r.
B.R. and popn.: D.R. const.	·1308 ± ·0209
D.R. and popn.: B.R. const.	·1474 ± ·0210
B.R. and D.R.: popn. const.	·0577 ± ·0213

Absolute numbers—

	Means and standard deviations.
Mean births	125·2653
S.D. „	62·6930
Mean deaths	73·8138
S.D. „	41·0272
Mean population	5092·0921
S.D. „	2290·1878

Correlation—

	r.	η.	Geometric mean.	Linearity test.*
Births and population	·9354 ± ·0027	{·9368 ·9509}	·9438	{1·200 4·008
Deaths and population	·8622 ± ·0055	{·8646 ·9240}	·8938	{1·509 7·788
Births and deaths	·8283 ± ·0067	{·8871 ·8469}	·8667	{7·445 4·138

Partial correlation—

Births and population: deaths const.	·7793 ± ·0084
Deaths and population: births const.	·4414 ± ·0172
Births and deaths: popn. const.	·1218 ± ·0210

* See Table 15.

TABLE 12.—*The curve-fitting constants* (1,000 *sub-districts*).

	μ_2.	μ_3.	μ_4.	β_1.	β_2.	Skewness.
Population	29 ·9871	28 ·6109	891 ·1887	·0886	2 ·0233	1 ·2770
Births	39 ·1825	142 ·4337	4091 ·8552	·3372	2 ·6652	·7144
Deaths................	21 ·2325	391 ·5979	22459 ·2957	16 ·0205	49 ·8188	·7342
Birth rates	14 ·7695	31 ·7140	1286 ·4067	·3122	5 ·8972	·1355
Death rates........	18 ·2788	613 ·0506	43814 ·7836	61 ·5392	131 ·1374	1 ·8963

TABLE 13.—*Planarity tests for* 1,000 *sub-districts.*

Rates—	H ("solid η").	$R_{1\cdot 23}$.
Death rates upon birth rates and population	·8088	·1521 ± ·0208
Population upon birth rates and death rates...........	·4929	·2035 ± ·0204
Absolute numbers—		
Deaths upon births and population	·9384	·7825 ± ·0083
Population upon births and deaths...........................	·9759	·9453 ± ·0023

TABLE 14.—*Planarity tests for* 999 *sub-districts.*

Rates—	H ("solid η").	$R_{1\cdot 23}$.
Death rates upon birth rates and population	·6796	·1666 ± ·0207
Absolute numbers—		
Deaths upon births and population...........................	·9213	·8644 ± ·0054

TABLE 15.—*Correlation constants within the arrays of population (births and deaths in 1,000 English registration sub-districts).*

Limits of population within the array.	Number of observations.	Rates.					Absolute numbers.				
		Number of arrays (κ).	r.	η.*	$\sqrt{\frac{\kappa-1}{N}}$.	Linearity test. $\frac{1}{2}\sqrt{\eta^2-r^2}\times\frac{\sqrt{N}}{\cdot 67449}$.	Number of arrays.	r.	η.*	$\sqrt{\frac{\kappa-1}{N}}$.	Linearity test. $\frac{1}{2}\sqrt{\eta^2-r^2}\times\frac{\sqrt{N}}{\cdot 67449}$.
1,000– 1,500	25	16	−·0434 ± ·1346	·9008	·7746	3·3349	4	−·1729 ± ·1309	·3171	·3464	·9853
1,500– 2,000	41·5	18	·1930 ± ·1008	·5056	·6400	2·2319	5	·1281 ± ·1030	·2824	·3105	1·2017
2,000– 2,500	67·5	18	−·1048 ± ·0812	·6251	·5018	3·7530	5	−·1041 ± ·0812	·2272	·2434	1·2299
2,500– 3,000	76·5	19	·0904 ± ·0765	·3036	·4851	1·8790	7	·1986 ± ·0741	·2718	·2801	1·2039
3,000– 3,500	95·5	17	·0445 ± ·0689	·3843	·4093	2·7649	7	·1451 ± ·0676	·3024	·2507	1·9223
3,500– 4,000	89	16	·0369 ± ·0714	·3701	·4105	2·5756	7	·0750 ± ·0711	·2000	·2596	1·2968
4,000– 4,500	69	12	−·1002 ± ·0804	·3450	·3993	2·0328	7	−·0883 ± ·0805	·3140	·2949	1·8481
4,500– 5,000	55	15	−·0461 ± ·0908	·7439	·5045	4·0819	8	−·0247 ± ·0909	·3555	·3568	1·9164
5,000– 5,500	68	14	−·0859 ± ·0812	·5575	·4372	3·3671	9	−·0451 ± ·0816	·5472	·3208	3·3335
5,500– 6,000	64	18	·0100 ± ·0843	·3419	·5154	2·0269	10	·0440 ± ·0841	·2973	·3750	1·7434
6,000– 6,500	58·5	19	−·2020 ± ·0846	·7158	·5547	3·7897	14	−·1961 ± ·0848	·6122	·4714	3·2881
6,500– 7,000	47·5	17	·3320 ± ·0870	·5299	·5804	2·1100	14	·3378 ± ·0867	·6629	·5231	2·9138
7,000– 7,500	49	14	·1265 ± ·0948	·5226	·5151	2·6310	12	·1031 ± ·0953	·2890	·4738	1·4012
7,500– 8,000†	52	15	−·3495 ± ·0821	·9778	·5189	4·8815	13	−·3533 ± ·0819	·9790	·4804	4·8805
8,000– 8,500‡	45	17	·3713 ± ·0867	·9057	·5963	4·1081	14	·3427 ± ·0887	·6820	·5375	2·9321
8,500– 9,000	41	17	−·0334 ± ·1052	·8733	·6247	4·1423	15	−·0379 ± ·1052	·8414	·5843	3·9899
9,000– 9,500	31	10	·2698 ± ·1123	·5841	·5388	2·1383	12	·3314 ± ·1078	·6709	·5957	3·0764
9,500–10,000	25	12	·4254 ± ·1105	·8637	·6633	2·7860	12	·4758 ± ·1044	·8708	·6633	2·6703

* The correlation ratios were calculated only for death rates upon birth rates and deaths upon births respectively.

† Includes one registration sub-district with 745 deaths (D.R. 94·3 per 1,000; B.R. 14·6 per 1000).

‡ „ „ „ „ 385 deaths (D.R. 47·6 per 1,000; B.R. 34·4 per 1,000).

TABLE 16.—*Empirical coefficients of partial association based on the geometric means of the correlation basis.*

A. 1,000 *sub-districts.*

	Coeff.
Rates—	
Birth rates and population : death rates const.	·1443
Death rates and population : birth rates const.	·1582
Birth rates and death rates : popn. const.	·3128
Absolute numbers—	
Births and population : deaths const.	·8176
Deaths and population : births const.	·4510
Births and deaths : population const.	·0789

B. 999 *sub-districts.*

Rates—	
Birth rates and population : death rates const.	·1642
Death rates and population : birth rates const.	·1807
Birth rates and death rates : population const.	·2070
Absolute numbers—	
Births and population : deaths const.	·7561
Deaths and population : births const.	·1453
Births and deaths : population const.	·1563

TABLE 17.—*A comparison between the correlation constants obtained when using absolute numbers and indices in the case of 118 English towns.*

Variables.	Absolute numbers.			Indices.		
	r.	η.*	$\frac{1}{2}\sqrt{\eta^2-r^2}\times\frac{\sqrt{N}}{\cdot 67449}$.	r.	η.*	$\frac{1}{2}\sqrt{\eta^2-r^2}\times\frac{\sqrt{N}}{\cdot 67449}$.
(1) *Crude values.*						
Cancer and diabetes	+ ·8925	·9174	1·710	+ ·3564	·4989	2·731
	± ·0126	·9094	1·405	± ·0542	·3892	1·259
Cancer and population	+ ·9423	·9511	1·044	+ ·0430	·2364	1·872
	+ ·0070	·9526	1·273	± ·0620	·3681	2·944
Diabetes and Population	+ ·8820	·9003	1·454	+ ·0031	·2166	1·744
	± ·0138	·9080	1·736	± ·0621	·1435	1·156
Cancer and diabetes, population constant.	+ ·3892	....	....	+ ·3566	....	....
	± ·0527	....	....	± ·0542	....	....
(2) *Corrected values.*						
Cancer and diabetes	+ ·8799	·9048	1·697	+ ·0475	·3569	2·849
	± ·0140	·9163	2·059	± ·0619	·2644	2·095
Cancer and population	+ ·9728	·9795	·924	+ ·1218	·2694	1·935
	± ·0033	·9782	·829	± ·0529	·4466	3·460
Diabetes and population	+ ·8909	·9106	1·520	+ ·0462	·2106	1·655
	± ·0128	·9276	2·090	± ·0620	·4264	3·413
Cancer and diabetes, population constant.	+ ·1259	....	....	+ ·0438	....	....
	± ·0611	....	....	± ·0620	....	....

* The two values correspond to the two regressions; that of x upon y and of y upon x.